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# Gauge link structure in quark–quark correlators in hard processes

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## Abstract

Distribution functions in hard processes can be described by quark–quark correlators, nonlocal matrix elements of quark fields. Color gauge invariance requires inclusion of appropriate gauge links in these correlators. For transverse momentum dependent distribution functions, in particular important for describing  $T$ -odd effects in hard processes, we find that new link structures containing loops can appear in Abelian and non-Abelian theories. In transverse moments, e.g., measured in azimuthal asymmetries, these loops may enhance the contribution of gluonic poles. Some explicit results for the link structure are given in high-energy lepton production and hadron–hadron scattering.

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## 1. Introduction

In this Letter we discuss the issue of color gauge invariance in bilocal operator matrix elements off the light-cone [1]. Such matrix elements are relevant in hard processes in which also transverse momenta of partons play a role such as semi-inclusive deep inelastic scattering (SIDIS) or the Drell–Yan process (DY). In these two cases the quark correlation functions that appear at leading order in an expansion in the inverse hard scale turn out to have a different gauge link structure. Upon integration over transverse momenta the difference in the gauge link structure does not matter, but for the transverse moments it does [2–5]. For the lowest transverse moments obtained from transverse momentum dependent distribution functions the difference corresponds to a time-reversal odd gluonic pole matrix element [6].

In the tree-level contributions to SIDIS and DY one was in essence dealing with diagrams with a single ‘active’ quark. In this Letter we consider more general situations, but in order to study the basic features we simplify the discussion by first considering QED-like hard interactions between the quarks. Complications that arise in QCD

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by the explicit presence of gluons will be discussed at the end, but the full treatment will be done as part of yet to come applications to hard QCD processes.

## 2. Gauge link structures in hard processes

In electroweak hard scattering processes such as deep inelastic scattering and the Drell–Yan process, the underlying hard processes  $\gamma^* + q \rightarrow q$  and  $\bar{q} + q \rightarrow \gamma^*$  involve a single quark. In these processes a hard scale  $Q$  is set by the virtuality of the photon. The transition hadron  $\rightarrow$  quark is described by the correlator

$$\Phi_{ij}(p, P) = \int \frac{d^4\xi}{(2\pi)^4} e^{ip \cdot \xi} \langle P | \bar{\psi}_j(0) \psi_i(\xi) | P \rangle, \quad (1)$$

where depending on the process the nonlocality is limited. Letting the soft quark and hadron momenta determine the lightcone direction  $n_+$ , one integrates over the quark momentum components  $p^- = p \cdot n_+$  and  $p_T$  in inclusive deep inelastic scattering (DIS), restricting the nonlocality to the lightcone [7–9]. In the case that more external momenta are measured, implying that other directions can be observed, one integrates only over the quark momentum component  $p^-$ , restricting the nonlocality to the light-front [10–12]. Examples are 1-particle inclusive or semi-inclusive deep inelastic scattering (SIDIS) and the Drell–Yan (DY) process, both of which involve two hadrons. In that case one has to deal with transverse momentum dependent distribution functions. Lightlike vectors  $n_+$  and  $n_-$  are introduced for convenience. They satisfy  $n_- \cdot n_+ = 1$  and are set by the external momenta. They define lightcone momenta  $a^\pm = a \cdot n_\mp$  and the transverse projector  $g_T^{\mu\nu} = g^{\mu\nu} - n_+^\mu n_-^\nu$ . Besides the correlator describing the hadron  $\rightarrow$  quark transition, correlators  $\bar{\Phi}(p, P)$  for the hadron  $\rightarrow$  antiquark and correlators  $\Delta(p, P)$  or  $\bar{\Delta}(p, P)$  for quark or antiquark  $\rightarrow$  hadron transitions can be written down [7,8].

Diagrams with additional gluons emerging from the soft hadronic parts are described by quark–gluon correlators

$$\Phi_{Aij}^\alpha(p, p - p_1, P) = \int \frac{d^4\xi}{(2\pi)^4} \frac{d^4\eta}{(2\pi)^4} e^{ip \cdot \xi} e^{ip_1 \cdot (\eta - \xi)} \langle P | \bar{\psi}_j(0) A^\alpha(\eta) \psi_i(\xi) | P \rangle. \quad (2)$$

In leading order of the expansion in inverse powers of the hard scale  $Q$ , one needs to include the  $\Phi_A^+$  correlators involving  $A^+ = A \cdot n_-$  (longitudinal) gluons that are collinear with their parent hadron. Together with multi-gluon correlators they produce gauge links along the  $n_-$  direction, connecting the points 0 and  $\xi$  to lightcone infinity [13]. Of the correlators involving transverse gluon fields, leading parts emerge that involve gluon fields at lightcone  $\pm\infty$ , which in combination with the links along  $n_-$  complete the gauge links between the points 0 and  $\xi$  (see Fig. 1) [3]. The resulting gauge links can be included in the correlator in Eq. (1), making it explicitly gauge invariant. The link structure, arising in the off-collinear situation in which one does not integrate over all transverse momenta of the quarks, turns out to have a different path structure for SIDIS as compared to Drell–Yan. In the case of SIDIS they come from  $A^+$  gluons coupling to an outgoing quark, while in DY they come from  $A^+$  gluons coupling to an incoming antiquark. The resulting path structures of the link operators are shown in Fig. 1.

In this Letter we present, in leading order of the inverse hard scale, the calculation of the gauge link structure in more complex hard processes. The hard subprocesses that we will consider in this section are quark–quark scattering and antiquark–quark scattering, which we will take as colorless interactions for the sake of simplicity. This suffices to illustrate the appearance of additional new structures in the gauge links.

The starting point in the calculation of high-energy hadronic scattering cross sections is the assumption that at tree-level it can be written as a convolution of hard, perturbatively calculable, partonic scattering processes with (the aforementioned) correlation functions describing the hadron  $\rightarrow$  quark or quark  $\rightarrow$  hadron transitions [14–16]. For example, the tree-level contribution to the cross section of the high-energy hadronic scattering process  $h_A + h_B \rightarrow h_C + h_D + X$  (see Fig. 2) involving the amplitude  $M$  for the hard quark–antiquark scattering process

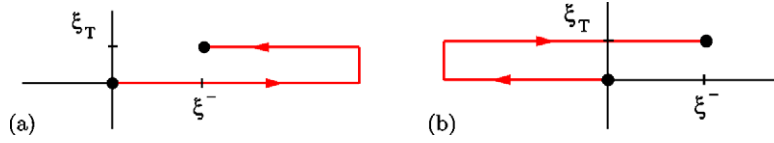


Fig. 1. The gauge link structure in the quark–quark correlator  $\Phi$  in SIDIS (a) and DY (b), respectively.

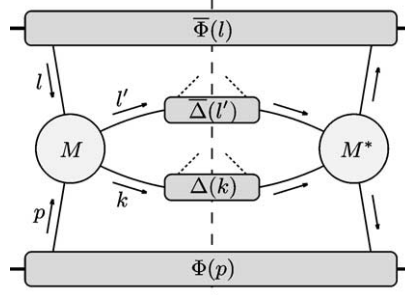


Fig. 2. Decomposition of the cross section for two-hadron production in hadron–hadron scattering with a hard antiquark–quark subprocess.

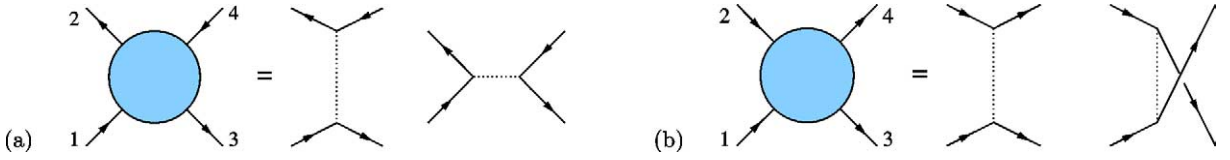


Fig. 3. (a) The  $t$ - and  $s$ -channel contributions to quark–antiquark scattering ( $q_1 + \bar{q}_2 \rightarrow q_3 + \bar{q}_4$ ). (b) The  $t$ - and  $u$ -channel contributions to quark–quark scattering ( $q_1 + q_2 \rightarrow q_3 + q_4$ ).

$q_1(p) + \bar{q}_2(l) \rightarrow q_3(k) + \bar{q}_4(l')$  is written as

$$\sigma \propto \int d^4p d^4k d^4l d^4l' \Phi(p) \otimes \bar{\Phi}(l) \otimes |M(p, k, l, l')|^2 \otimes \Delta(k) \otimes \bar{\Delta}(l'). \quad (3)$$

The hard partonic scattering may contain several contributions, see Fig. 3a. The correlators  $\Phi$ ,  $\bar{\Phi}$ ,  $\Delta$  and  $\bar{\Delta}$  describe the soft parts connecting the quarks in the hard process to the hadrons involved in the physical (partly inclusive) process. The convolution in Eq. (3) indicates that one needs specific Dirac tracing, depending on the particular forms of  $M$ .

As an illustration of Eq. (3), let us consider the situation where the incoming quark and antiquark have different flavors, in which case one only has the  $t$ -channel contribution in the hard process in Fig. 3a. Because we want to illustrate the involvement of another strongly interacting particle in the hard process, it is convenient to keep the hard process itself colorless. Therefore, we consider the exchange of a photon as indicated in Fig. 4a. The expression for the scattering cross section is

$$\sigma \propto \int d^4p d^4k d^4l d^4l' \delta^4(p + q - k) \left\{ \left( \frac{1}{q^2} \right)^2 \text{Tr}[\Phi(p) \gamma^\mu \Delta(k) \gamma^\nu] \text{Tr}[\bar{\Phi}(l) \gamma_\nu \bar{\Delta}(l') \gamma_\mu] \right\}, \quad (4)$$

where  $q = l' - l$ . To obtain the link structure of the  $\Phi(p)$  correlator in Eq. (3), we consider all the (colorless) gluon insertions coming from the lower blob. All other soft correlators in the hadronic scattering process also get specific gauge link structures from their respective longitudinal gluons coupling to the hard part. Introducing a set of light-like vectors  $n_+$  and  $n_-$ , such that  $n_+$  is proportional to the momentum of the lower blob and  $n_- \cdot n_+ = 1$ , we find that the single  $A^+ = A \cdot n_-$  gluon insertions on the l.h.s. of the cut lead to the additional contributions in

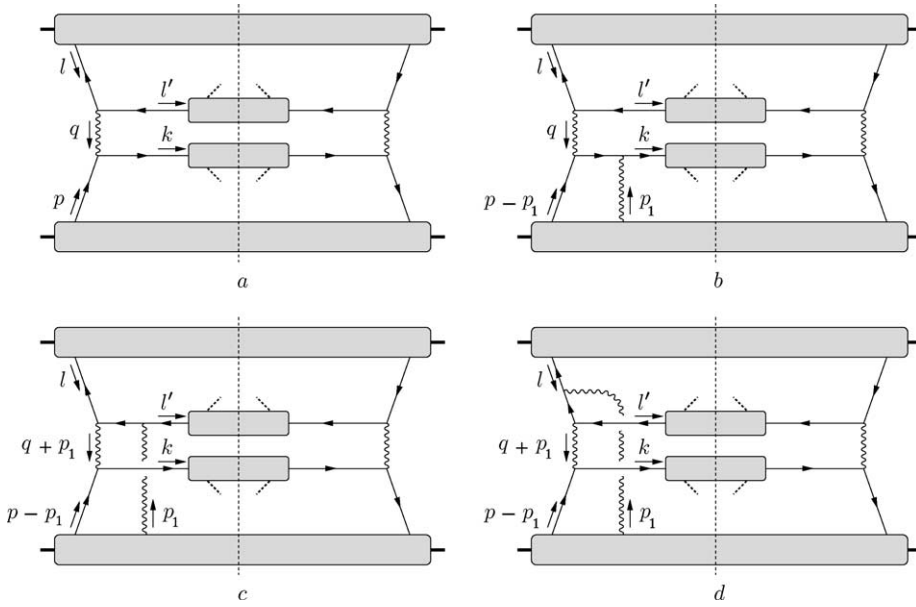


Fig. 4. Treelevel diagram and insertions of longitudinal gauge bosons.

the cross section in Fig. 4b–d

$$\begin{aligned}
 \sigma \propto & \int d^4 p d^4 k d^4 l d^4 l' \delta^4(p + q - k) \frac{1}{q^2} \left\{ \frac{1}{q^2 + i\epsilon} \text{Tr}[\Phi(p) \gamma^\mu \Delta(k) \gamma^\nu] \text{Tr}[\bar{\Phi}(l) \gamma_\nu \bar{\Delta}(l') \gamma_\mu] \right. \\
 & + g_2 \int d^4 p_1 \frac{1}{q^2 + i\epsilon} \text{Tr} \left[ \Phi_A^\alpha(p, p - p_1) \gamma^\mu \Delta(k) (i\gamma_\alpha) \frac{i(\not{k} - \not{p}_1)}{(k - p_1)^2 + i\epsilon} \gamma^\nu \right] \text{Tr}[\bar{\Phi}(l) \gamma_\nu \bar{\Delta}(l') \gamma_\mu] \\
 & + g_1 \int d^4 p_1 \frac{1}{(q + p_1)^2 + i\epsilon} \text{Tr}[\Phi_A^\alpha(p, p - p_1) \gamma^\mu \Delta(k) \gamma^\nu] \text{Tr} \left[ \bar{\Phi}(l) \gamma_\nu \frac{i(-\not{l}' + \not{p}_1)}{(l' - p_1)^2 + i\epsilon} (i\gamma_\alpha) \bar{\Delta}(l') \gamma_\mu \right] \\
 & + g_1 \int d^4 p_1 \frac{1}{(q + p_1)^2 + i\epsilon} \text{Tr}[\Phi_A^\alpha(p, p - p_1) \gamma^\mu \Delta(k) \gamma^\nu] \\
 & \left. \times \text{Tr} \left[ \bar{\Phi}(l) (i\gamma_\alpha) \frac{i(-\not{l} - \not{p}_1)}{(l + p_1)^2 + i\epsilon} \gamma_\nu \bar{\Delta}(l') \gamma_\mu \right] \right\}, \quad (5)
 \end{aligned}$$

where  $p$  and  $p_1$  are collinear with  $n_+$ . For ease of distinguishing the involved fermions, the coupling constants of the longitudinal gauge particles are indicated with  $g_1$  and  $g_2$ . Investigating the analytic structure in  $p_1^+$ , one finds that the poles at  $p_1^+ \neq 0$  cancel each other. This is analogous to Ward identities, where consecutive insertions take care of all the cancellations of the poles at  $p_1^+ \neq 0$ . Making use of the fact that for the leading contributions in the soft correlators  $\Phi(p) \propto \not{p}$ ,  $\Delta(k) \propto \not{k}$ ,  $\bar{\Phi}(l) \propto \not{l}$  and  $\Delta(l') \propto \not{l}'$  [17], the final result for the scattering cross section is

$$\begin{aligned}
 \sigma \propto & \int d^4 p d^4 k d^4 l d^4 l' \delta^4(p + q - k) \left( \frac{1}{q^2} \right)^2 \left\{ \text{Tr}[\Phi(p) \gamma^\mu \Delta(k) \gamma^\nu] \text{Tr}[\bar{\Phi}(l) \gamma_\nu \bar{\Delta}(l') \gamma_\mu] \right. \\
 & \left. - g_2 \int d^4 p_1 \frac{1}{-p_1^+ + i\epsilon} \text{Tr}[\Phi_A^\alpha(p, p - p_1) \gamma^\mu \Delta(k) \gamma^\nu] \text{Tr}[\bar{\Phi}(l) \gamma_\nu \bar{\Delta}(l') \gamma_\mu] \right\}
 \end{aligned}$$

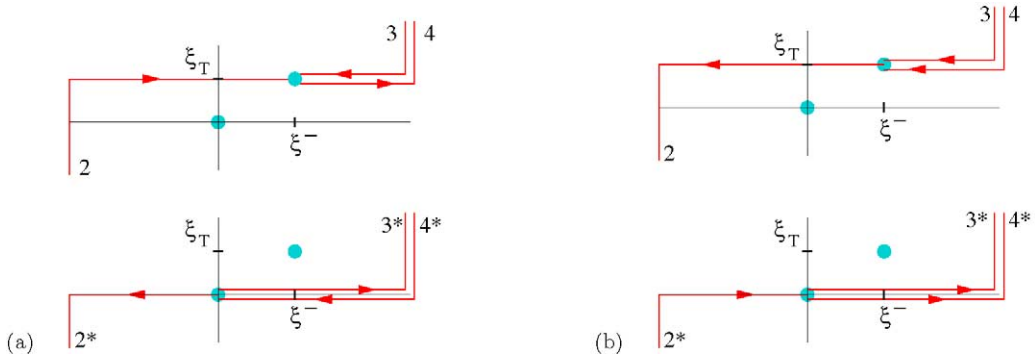


Fig. 5. The basic gauge links for distribution functions coming from the soft correlator for quark 1 that arise from  $A^+$ -gluons interacting with the other initial- or final-state fermion legs in the amplitude (2, 3 and 4) and the conjugate amplitude ( $2^*$ ,  $3^*$  and  $4^*$ ) for (a) quark–antiquark scattering ( $q_1 + \bar{q}_2 \rightarrow q_3 + \bar{q}_4$ ) (b) quark–quark scattering ( $q_1 + q_2 \rightarrow q_3 + q_4$ ).

$$\begin{aligned}
 & + g_1 \int d^4 p_1 \frac{1}{-p_1^+ + i\epsilon} \text{Tr}[\Phi_A^\alpha(p, p - p_1) \gamma^\mu \Delta(k) \gamma^\nu] \text{Tr}[\bar{\Phi}(l) \gamma_\nu \bar{\Delta}(l') \gamma_\mu] \\
 & + g_1 \int d^4 p_1 \frac{1}{p_1^+ + i\epsilon} \text{Tr}[\Phi_A^\alpha(p, p - p_1) \gamma^\mu \Delta(k) \gamma^\nu] \text{Tr}[\bar{\Phi}(l) \gamma_\nu \bar{\Delta}(l') \gamma_\mu] \Big\}. \quad (6)
 \end{aligned}$$

The contributions found in Eq. (6) represent the first terms of gauge links  $U_{g_2}(\infty^-, \xi_T; \xi^-, \xi_T)$ ,  $U_{g_1}(\xi^-, \xi_T; \infty^-, \xi_T)$ , and  $U_{g_1}(-\infty^-, \xi_T; \xi^-, \xi_T)$ , respectively. They are of the form

$$U_g(a; b) = \mathcal{P} \exp \left( -ig \int_a^b dx \cdot A(x) \right), \quad (7)$$

with the integration along a straight line between  $a$  and  $b$ . The link structure arising in this way from the insertions of longitudinal gluons coming from  $\Phi(p)$  (Fig. 4b–d) in the hard amplitude  $q_1 \bar{q}_2 \rightarrow q_3 \bar{q}_4$  is indicated in Fig. 5a. The same figure also gives the results for the insertions in the conjugate diagram, which can be handled in the same way and lead to the links connecting the point 0 in the correlation function to lightcone infinity. The first order calculations presented here explicitly, can be extended to include all longitudinal gluon insertions to which we, without giving the derivation, have added the transverse gauge link pieces. These transverse fields at infinity emerge, in the same way as for the simple processes with just a single quark line, as boundary terms that also need to be subtracted from transverse gluon correlators  $\Phi_A^\alpha$  to obtain gauge invariant correlators in terms of the field strength tensor [6]. For a hard quark–quark scattering amplitude  $q_1 q_2 \rightarrow q_3 q_4$  the resulting link structures are given in Fig. 5b.

Taking all the insertions of longitudinal gluons into account, as in Eq. (5), we have found that this expression reduces to Eq. (3) with the correlator  $\Phi(p)$  now containing a link connecting the quark fields at 0 and  $\xi$ . Three combinations of links appear

$$\mathcal{U}_g^{[-]} = U_g(0; -\infty^-, 0_T) U_g(-\infty^-, 0_T; -\infty^-, \xi_T) U_g(\infty^-, \xi_T; \xi), \quad (8)$$

$$\mathcal{U}_g^{[+]} = U_g(0; \infty^-, 0_T) U_g(\infty^-, 0_T; \infty^-, \xi_T) U_g(\infty^-, \xi_T; \xi), \quad (9)$$

$$\mathcal{U}_g^{[\square]} = \mathcal{U}_g^{[+]} \mathcal{U}_g^{[-]\dagger}, \quad (10)$$

where the latter constitutes a counterclockwise (Wilson) loop from the point 0 via lightcone infinity to  $\xi$  and back via negative lightcone infinity to 0. The result for  $h_A + h_B \rightarrow h_C + h_D + X$  with the quark–antiquark subprocess is a gauge link insertion  $\mathcal{U}_{g_2}^{[+]} \text{Tr}(\mathcal{U}_{g_1}^{[\square]\dagger})$ , shown in Fig. 6a. The trace operation introduced in this figure will become relevant when we take the full color structure of the inserted gluons into account.

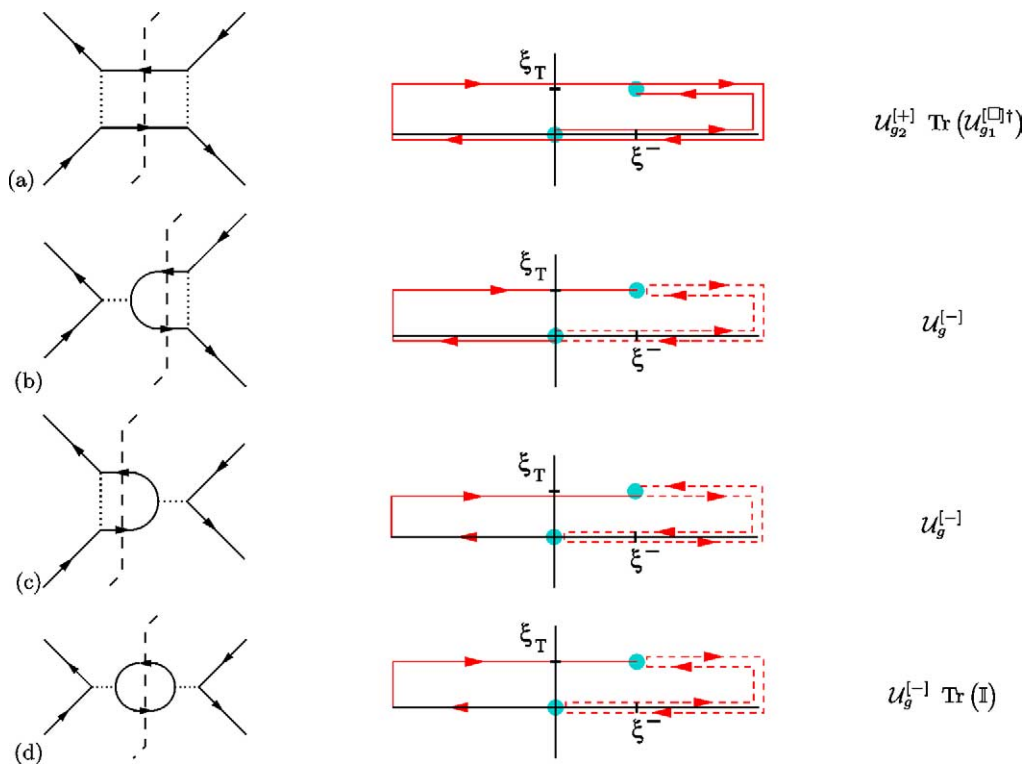


Fig. 6. Contributions to the cross section of antiquark–quark scattering and the corresponding gauge link structures. In the contributions b–d one has  $g_1 = g_2 = g$ .

An interesting case to consider is when  $q_1$  and  $\bar{q}_2$  are each others antiparticles. Besides the  $t$ -channel contribution which we have treated above, there is also the  $s$ -channel quark–antiquark annihilation contribution (see Fig. 3a). The link structure for the  $s$ -channel amplitude and its conjugate are, just as the  $t$ -channel contribution, given by the gauge lines in Fig. 5a. Including interference terms, four contributions arise in the cross section. We observe that in these terms the correlation function  $\Phi(p)$  appears with different gauge link structures as indicated in Fig. 6b–d. In the terms of the scattering cross section that contain an  $s$ -channel amplitude, we again get the product of the link operators  $\mathcal{U}_{g_2}^{[+]}$  and  $\mathcal{U}_{g_1}^{[\square]\dagger}$ , but in these terms we only have a single coupling constant  $g_1 = g_2 = g$  and the two link operators add up to the link structure  $\mathcal{U}_g^{[-]}$ . We note that the result in Fig. 6d actually involves  $\text{Tr}(\mathcal{U}_g^{[+]} \mathcal{U}_g^{[+]\dagger})$ , i.e., the trace of the unity operator in the charge space, which becomes relevant in case the color structure is considered.

If we take the hard process  $M(p, k, l, l')$  in Fig. 2 to represent quark–quark scattering, the gauge links arising from the insertions of collinear gluons in the amplitude and its conjugate (Fig. 5b) combine into one or (in case of identical quarks) four possible link structures in the cross section, as summarized in Fig. 7. Again we get the product of the link operators  $\mathcal{U}_{g_2}^{[+]}$  and  $\mathcal{U}_{g_1}^{[\square]}$ , where  $\mathcal{U}_{g_1}^{[\square]}$  is traced if the amplitudes on both sides of the cut are each others Hermitian conjugates. For the interference terms in quark–quark scattering, however, the two operators do not add up to a simple link operator such as  $\mathcal{U}_g^{[-]}$ .

The loop that we have found in the gauge link disappears upon integration of the matrix elements over transverse momentum. Therefore, it only affects those processes where one is sensitive to the transverse momentum of the quarks inside the nucleon. These involve transverse moments  $\Phi_j^\alpha(x)$ , defined as transverse momentum weighted integrals of the  $p_T$ -dependent distribution functions. In these moments the loop will contribute through a gluonic

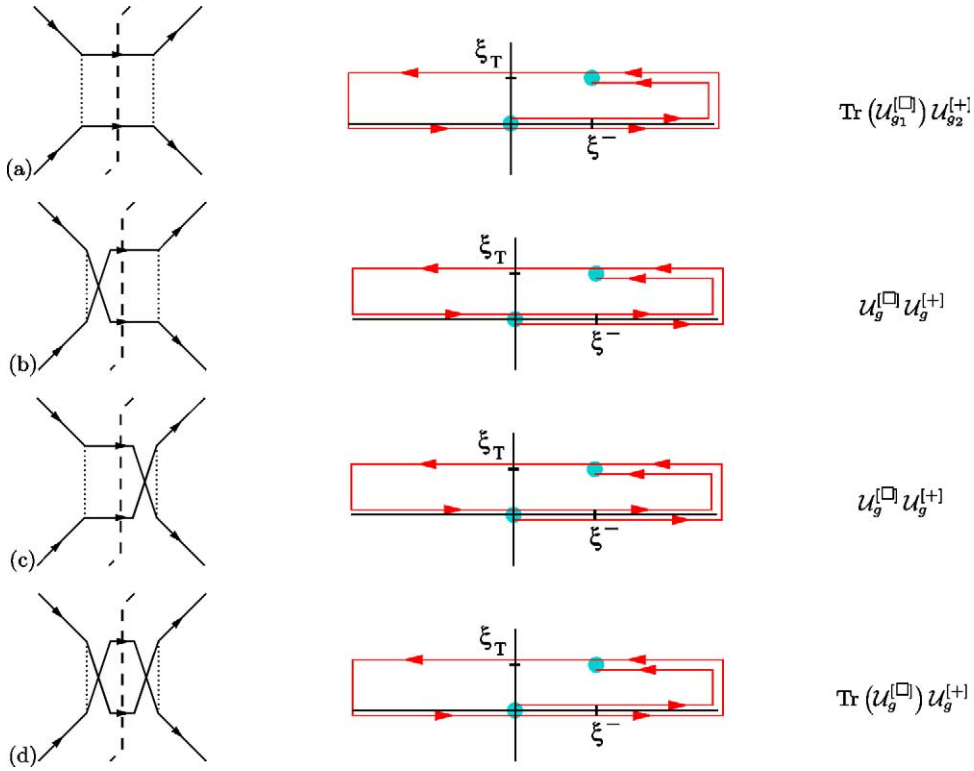


Fig. 7. Contributions to the cross section of quark–quark scattering and the corresponding gauge link structures. In the contributions b–d one has  $g_1 = g_2 = g$ .

pole matrix element, responsible for time-reversal odd distribution functions, such as the Sivers function [18,19]. For example, for unpolarized quarks in SIDIS one has [6]

$$\begin{aligned}
 & \text{Tr}[\Phi_\partial^{[+]}(x)\gamma^+] \\
 &= \int d^2 p_T p_T^\alpha \int \frac{d^2 \xi_T d\xi^-}{(2\pi)^3} e^{ip \cdot \xi} \langle P, S | \bar{\psi}(0) \gamma^+ \mathcal{U}_g^{[+]}(0; \xi) \psi(\xi) | P, S \rangle |_{\xi^+ = 0} \\
 &= \frac{g}{2} \int \frac{d\xi^-}{2\pi} e^{ip^+ \xi^-} \langle P, S | \bar{\psi}(0) \gamma^+ \int_{-\infty}^{\infty} d\eta^- \mathcal{U}(0; \eta^-) G^{+\alpha}(\eta^-) \mathcal{U}(\eta^-; \xi^-) \psi(\xi) | P, S \rangle |_{\xi^+ = \xi_T = 0}, \quad (11)
 \end{aligned}$$

(only the  $T$ -odd part). Transverse moments appearing in processes involving hard quark–quark scattering yield different results. Taking, for instance, the link structure in Fig. 7b gives

$$\begin{aligned}
 & \int d^2 p_T p_T^\alpha \int \frac{d^2 \xi_T d\xi^-}{(2\pi)^3} e^{ip \cdot \xi} \langle P, S | \bar{\psi}(0) \gamma^+ \mathcal{U}_g^{[\square]} \mathcal{U}_g^{[+]}(0; \xi) \psi(\xi) | P, S \rangle |_{\xi^+ = 0} \\
 &= 3 \frac{g}{2} \int \frac{d\xi^-}{2\pi} e^{ip^+ \xi^-} \langle P, S | \bar{\psi}(0) \gamma^+ \int_{-\infty}^{\infty} d\eta^- \mathcal{U}(0; \eta^-) G^{+\alpha}(\eta^-) \mathcal{U}(\eta^-; \xi^-) \psi(\xi) | P, S \rangle |_{\xi^+ = \xi_T = 0}, \quad (12)
 \end{aligned}$$



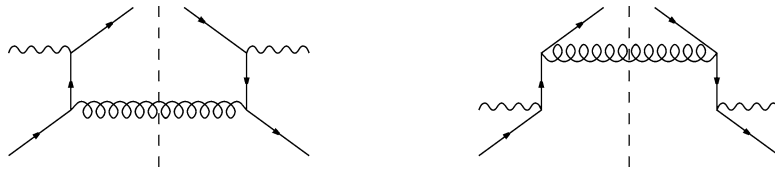


Fig. 8. Amplitudes which contribute to the SIDIS cross section in which a hard gluon is radiated.



Fig. 9. Amplitudes which contribute to the DY cross section in which a hard gluon is radiated.

where one finds that the difference is again a gluonic pole matrix element, but with a different strength. This strength is set by the link structure and thus by the process, a feature that also appears in non-Abelian theories, as we will see in the next section.

### 3. Link structures in QCD processes

Although we have made various simplifying assumptions in the processes that we have considered in the previous section, it illustrates the appearance of open and closed integration paths in the gauge link in the correlator  $\Phi$ , which we will also find in QCD. In the Abelian examples given above the only particles carrying charge were the fermions, hence the charge tracing followed the Dirac tracing. In the non-Abelian case such as QCD, this is in general no longer the case. In the previous section, we already indicated the charge flow and contractions via the traces in Figs. 6 and 7, which is particularly relevant in QCD. The non-Abelian loops are not gauge invariant by themselves, but only together with the quark fields in the correlator or when they are color-traced. In the calculation of several QCD processes we find that the sum of all gluon insertions to a single amplitude again does not have any poles at  $p_1^+ \neq 0$ .

The first example that we will give here is SIDIS at large  $p_T$ , for which the leading hard parts are given in Fig. 8. One can distinguish this process from ordinary SIDIS by looking at two jet production. The longitudinal gluons coming from the incoming proton are collected into links. These gluons couple to the radiated gluon, the fragmenting quark and the off-shell quark in the hard process. Since there are no incoming lines to which the longitudinal gluon can couple, loops will not appear. A calculation taking all possible insertions into account confirms this and shows that the link structure is similar to that in ordinary SIDIS

$$\text{large } p_T \text{ SIDIS: } \frac{T_F N_c}{C_F} \mathcal{U}^{[+]} \frac{1}{N_c} \text{Tr}^C [\mathcal{U}^{[+]\dagger} \mathcal{U}^{[+]}] - \frac{T_F}{C_F N_c} \mathcal{U}^{[+]} = \mathcal{U}^{[+]}, \quad (13)$$

where the symbol  $\text{Tr}^C$  is used to distinguish color tracing from Dirac tracing and  $T_F = 1/2$  and  $C_F = T_F(N_c^2 - 1)/N_c = 4/3$  in  $SU(3)$ . The weighted transverse moment appearing in an appropriate azimuthal asymmetry is therefore  $\Phi_\theta^{[+]\alpha}$ .

The second process considered here is DY at large  $p_T$ , where the situation is more complicated. In simple DY there is only one (incoming) line to which longitudinal gluons can couple, which leads to a  $\mathcal{U}^{[-]}$ -link. However, if a hard gluon is radiated (see Fig. 9), the link structure changes completely. The outgoing line together with the incoming line give in general the possible presence of a loop and this is exactly what occurs here. The calculation

leads to a link structure in the quark–quark correlator for the lower hadron of the form

$$\text{large } p_T \text{ DY: } \frac{T_F N_c}{C_F} \mathcal{U}^{[+]} \frac{1}{N_c} \text{Tr}^C [\mathcal{U}^{[\square]\dagger}] - \frac{T_F}{C_F N_c} \mathcal{U}^{[-]} = \frac{9}{8} \mathcal{U}^{[+]} \frac{1}{N_c} \text{Tr}^C [\mathcal{U}^{[\square]\dagger}] - \frac{1}{8} \mathcal{U}^{[-]}. \quad (14)$$

The explicit links and their differences only appear in the (unintegrated) transverse momentum dependent parton distributions. After integration over transverse momenta, the differences vanish in  $\Phi(x)$ . Weighing with  $p_T$  once, one obtains the transverse moments  $\Phi_\theta^\alpha(x)$  and, as we have seen in Eq. (12), these differences may lead to an enhancement of the gluonic pole contribution in the transverse moment. In QCD, however, this is not the case for traced loops. For a loop in the gauge link  $\text{Tr}^C [\mathcal{U}^{[\square]}]$ , one obtains a structure in the integrand of the form  $\text{Tr}^C [\mathcal{U} G^{+\alpha} \mathcal{U}]$ , which vanishes. Therefore the transverse moment appearing in large  $p_T$  DY has the link structure  $\frac{9}{8} \Phi_\theta^{[+]\alpha} - \frac{1}{8} \Phi_\theta^{[-]\alpha}$ . It differs from the ordinary DY process, where the transverse moment is  $\Phi_\theta^{[-]\alpha}$ .

We mention here two other examples, namely processes with quark–antiquark and quark–quark scattering as hard processes, relevant in processes like pion production in hadron–hadron scattering. Each of the soft parts in such a process gets a particular link structure. Considering for hadron–hadron scattering only the contribution coming from quark–antiquark scattering given in Fig. 6a, with in the  $t$ -channel now gluon exchange, one finds that all four link structures in Fig. 6 are involved due to the color-flow,

$$\text{Fig. 6a in QCD: } \frac{1}{8} \mathcal{U}^{[+]} \frac{1}{N_c} \text{Tr}^C [\mathcal{U}^{[\square]\dagger}] + \frac{7}{8} \mathcal{U}^{[-]}. \quad (15)$$

For quark–quark scattering one finds

$$\text{Fig. 7a in QCD: } \frac{5}{4} \frac{1}{N_c} \text{Tr}^C [\mathcal{U}^{[\square]\dagger}] \mathcal{U}^{[+]} - \frac{1}{4} \mathcal{U}^{[\square]} \mathcal{U}^{[+]}, \quad (16)$$

where the latter term is an example of an open loop, which, in analogy to Eq. (12), gives an enhancement of the gluonic pole contribution as compared to  $\mathcal{U}^{[+]}$ .

In the above examples we have only been concerned with the link structure of the correlator  $\Phi$ , which describes the embedding of an incoming quark. The link structure arose from longitudinal collinear gluons. In the same way the other correlators must be dealt with and will acquire particular link structures, which also may involve loops. One of those correlators, which describes the fragmentation of a quark in SIDIS at large  $p_T$ , receives, for instance, a loop contribution. Since there are two sources for  $T$ -odd effects on the fragmentation side, the transverse moment as given in Eqs. (11) and (12) consists in this case of two terms, one coming from gluonic poles and one from final state interactions. As argued in [6], the combination of these two mechanisms spoils the simple sign relation between unintegrated fragmentation functions appearing in SIDIS and electron–positron annihilation found in Ref. [20], which uses the model of Ref. [21].

## 4. Conclusions

In this Letter we have shown that loops can appear in link structures in the correlation functions used in hard processes. These link structures become relevant as soon as one is sensitive to the transverse momenta of the partons. The loops may contribute to the transverse moments as enhanced gluonic poles. For distribution functions, gluonic poles are the origin of the  $T$ -odd distribution functions appearing in single spin asymmetries. In order to show the appearance of loops we started with an Abelian version of QCD. However, also in non-Abelian theories, loops appear for which a few explicit results have been given.

The link structures can be derived by resumming all longitudinally polarized gluons, coming from a soft blob, into a gauge invariant correlation function. Although this seems to break universality, this needs not to be the case in  $p_T$ -integrated and  $p_T$ -weighted quantities, which lead to well-defined bilocal lightcone matrix elements. Nevertheless, in the study of factorization (see, e.g., [22,23]) and the evolution of transverse momentum dependent

distribution and fragmentation functions [24] one also needs to account for the structure and appearance of loops in the gauge links. In principle things look fine for SIDIS, because we found that one is always dealing with a  $\mathcal{U}^{[+]}$ -link, but in DY a gluon ladder leads to different link structure. Also for fragmentation in SIDIS one finds such differences.

We have discussed the link structures appearing in large  $p_T$  SIDIS and large  $p_T$  DY processes. We note that having found the link structures, there is still work to be done in finding out the specific observables in which one sees their effects. One needs an observable sensitive to the intrinsic transverse momentum of the partons, which is different from the large  $p_T$  [25].  $T$ -odd observables, such as the L–R asymmetry in  $pp^\uparrow \rightarrow \pi X$  [26–29], may play an important role here, as they are absent for integrated distribution functions at leading order.

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